Linear Bus Holding Model for Real-Time Traffic Network Control

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Linear Bus Holding Model for Traffic Network

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Abstract One of the most annoying problems in urban bus operations is *bus bunching*, which happens when two or more buses arrive at a stop nose to tail. Bus bunching reflects an unreliable service that affects transit operations by increasing passenger-waiting times. This work proposes a linear mathematical programming model that establishes bus holding times at certain stops along a transit corridor to avoid bus bunching. Our approach needs real-time input, so we simulate a transit corridor and apply our mathematical model to the data generated. Thus, the inherent variability of a transit system is considered by the simulation, while the optimization model takes into account the key variables and constraints of the bus operation. Most of the literature considers quadratic models that minimize passenger-waiting times, but they are harder to solve and therefore difficult to operate by real-time systems. On the other hand, our methodology reduces passenger-waiting times efficiently given our linear programming model, with the characteristic of applying control intervals just every 5 minutes.

Keywords Bus bunching · Holding times · Linear programming · Simulation

1 Introduction and problem description

The study of complex bus operating systems is usually divided in two main areas, *line planning* and *real-time control* (Ceder, 2007; Desaulniers and Hickman, 2007). The *line planning* process involves strategic, tactical and operational decisions. Strategic problems relate to long-term network design decisions. Tactical and operational decisions ultimately define the service offered to the public; for example, frequency of buses, definition of stops, bus timetabling,

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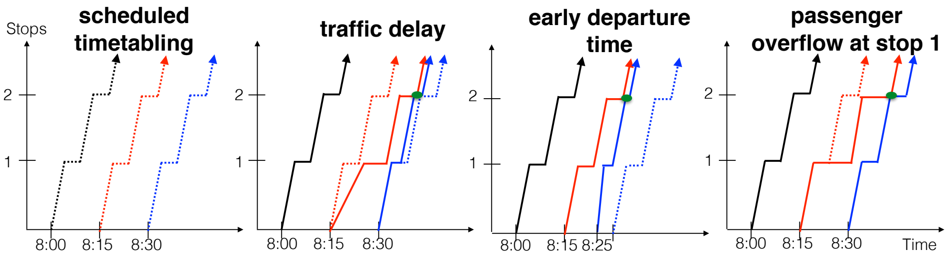


Fig. 1 Causes of bus bunching (modified from Ceder (2007)).

vehicle scheduling, driver scheduling, maintenance scheduling, among other problems. On the other hand, *real-time control* tries to maintain the bus system operational along the day in order to minimize passenger inconvenience (Desaulniers and Hickman, 2007).

Real-time control is hard because of the dynamics of the network or traffic situations. Although, bus frequency is planned for each stop in the network, changes in the passenger flow, traffic or even in the timetabling, produce perturbations that alter frequency plans and give rise to one of the most annoying problems in urban transportation operations, the *bus bunching problem (BBP)*. BBP happens when two or more buses arrive at a stop nose to tail, and it reflects an unreliable service that affects transit operations by increasing passenger-waiting times.

In Figure 1, we show the causes of bus bunching for a single bus line with three trips, which have the following timetable: 8:00, 8:15, and 8:30. For the four graphs, time is represented by the x-axis, while the first two stops are represented by the y-axis. The first graph shows how the planning should look like if everything were deterministic. We can see that the lines of the three trips are *parallel*, so the differences between their departure times (called headways) are of exactly 15 minutes. The second graph shows the perturbations that arise when a traffic delay hits the second trip between the depot and the first stop. The doted lines are the planned schedules, while the plain lines are the real executed delayed plans. Graph three represents bunching situations when the departure time of a trip is moved earlier, which happens to the third trip in this example. Finally, the fourth graph considers the case of passenger overflow. This graph shows that since there are more passengers at stop 1, the dwell time of the second bus at that stop will be longer. In other words, the second bus is taking passengers that would be normally assigned to the third bus. By the time the second and third bus arrive at stop 2, they are generating a bus bunching situation. BBP is one of the most common customer complaints in today's networks

On this work, we focus on providing solutions to the *bus bunching problem* (BBP) by maintaining *congruent headways*. Furthermore, we will show that maintaining congruent headways implicitly reduces passenger-waiting times. A headway is a quality measure given to the time difference between two consecutive buses. A bus line could have equally distant headways or different

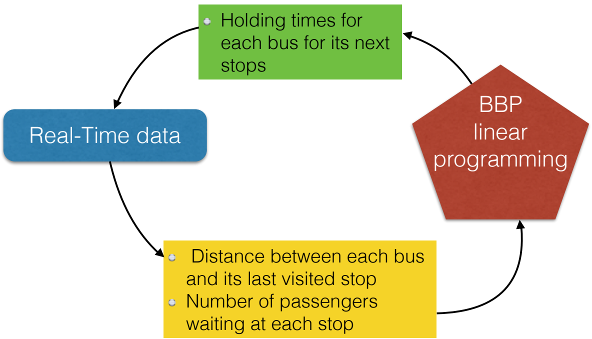


Fig. 2 Framework for interleaving Planning (BBP LP modeling) and Execution (real-time simulation)

ones for each pair of buses Ceder (2001); Ibarra-Rojas and Rios-Solis (2012); Ibarra-Rojas et al (2014). We say that *headways are congruent* if the real-time differences between buses are nearly identical to the originally planned. Headway congruence does not necessarily comply with planned timetables. Indeed, the time when a bus arrives at a stop may not be the planned one, but if the distance to his predecessor is almost the planned headway, then it will be a congruent headway. Congruent headways reflect a reliable service especially for the cases when timetables are not publically available.

Our methodology interleaves planning and execution to maintain congruent headways and solve BBP. During the planning phase, a linear programming model is built and solved to exactly determine the holding times of the buses at the stops. The execution phase, in our methodology, is simulated; it indicates, at every interval of time, the positions of the buses along a single corridor where only one line operates on a given frequency. In Figure 2, we can observe how planning and execution (i.e., simulation) interleave. Real-time (or simulated) data are acquired from the bus corridor to obtain the distance between each bus and its last visited stop, together with the number of passengers waiting at each stop. Then, these data are used to populate our linear programming model, which yields the optimal holding times for each bus in the corridor. Most of the works in the literature base their quality measure on the waiting times of the passengers, or the variance between the departure times of the buses, which can be modeled with quadratic functions that are harder to solve. Instead, by using a linear programming model, our methodology returns optimal solutions at a fraction of the cost of alternative approaches. Moreover, maintaining more stable headways reduces passenger-waiting times in the lines, as our experimental results will demonstrate.

The rest of this article is structured as follows. A brief revision of the state of the art is presented in Section 2. In Section 3, we present our new linear programming model inspired in earliness and tardiness penalties of just-in-time scheduling problems, which determines the optimal holding times of the buses at the stops. Then, Section 4 shows the efficiency of our model on a discrete event simulation of a single corridor. Finally, Section 5 presents our conclusions, and discusses open research questions that arise from this work.

2 State of the art research in real-time bus operations

Most of the literature related to real-time bus operations deals with models that have non-linear objective functions. Therefore, the holding times that each bus most be held at the stations are approximations. Work by Zhao et al (2003) minimizes the average waiting cost of passengers, including both off-bus and on-bus costs that are non-linear, when there is no capacity imposed to the buses. Eberlein et al (2001) minimize the variance between the departure times, which is a quadratic function, and therefore propose heuristic solutions. Sun and Hickman (2008) propose a convex quadratic programming problem to minimize the variance between the departure times. A closer work to ours is proposed by Ding and Chien (2001), since they consider the minimization of the total variance of headways between buses in all stops.

Daganzo (2009) and Daganzo and Pilachowski (2011) propose adaptive control schemes aiming to provide quasi-regular headways, while maintaining as high commercial speed as possible. In Daganzo and Pilachowski (2011) the authors continuously adjust bus cruising speed based on a cooperative two way based approach that considers the headways of the previous and posterior buses. Notice that these approaches require that the headways are equal between all the buses, which is not necessary in our case. Bartholdi III and Eisenstein (2012) abandon the idea of any *a priori* target headway, allowing headways to dynamically self-equalize by implementing a simple holding rule at a control point. It is worth noting that the aim of these studies is to maintain equally headways, but they are not apt for situations when the buses reach their capacities.

Our work deals with capacity on the vehicles as Zolfaghari et al (2004) do, where the authors minimize the waiting time of passengers at every stop by taking into account the variance between the departure times. These authors propose heuristics to circumvent the complexity of the proposed model. Puong and Wilson (2008) propose a non-linear mixed-integer linear programming for a real-time disruption response model with emphasis on the train holding strategy. In Delgado et al (2009) and Delgado et al (2012) the aim is to minimize the total waiting times experienced by passengers in the system using a quadratic model.

It can be noticed that there is not a bus holding strategy that considers a linear function as we do. Moreover, as introduced in this section, most of the current works in the literature focus in minimizing passenger-waiting times. Our work aims at maintaining congruent headways considering capacity on the vehicles, and in doing so, we expect to reduce passenger-waiting times in the bus corridors. We also improve the work of Delgado et al (2012) by reducing the number of variables in the model, and the number of times the model is used in real-time scenarios, obtaining exact solutions for the holding times.

Another advantage of our proposal is that it adapts easily to the cases where the headways are equal or different during different planning horizons along the day.

3 Methodology and approach

As mentioned earlier, the core of our methodology consists on interleaving planning and execution of the bus lines. The planning phase of our approach builds and solves efficiently an optimization model that is linear to maintain congruent headways along the bus line. Our model is used every given time interval[1] to decide how long the buses should be held at the bus stops. Our model requires a real-time data estimation of the state of the system to operate. Such data is provided by the execution phase, which in our case is supported via simulation. The simulation of the system provides data related to position of the buses, number of passengers aboard of each bus, and the number of passengers waiting at the stops to build our model.

More precisely, the Bus Bunching Problem (BBP) consists of K buses, each with its own capacity and speed that serve all S stops of a single bus corridor. We can see in Figure 3 that each bus k leaves the depot according to an established timetable, serving stops 1 to S before coming back to the depot where all remaining passengers must alight. Notice that overtaking is not permitted. We consider that travel times between stops, and λ_s (passengers arrival rate per minute) are deterministic during the period of interest. Moreover, each stop has a dwell time function depending linearly on the number of passengers that board ($boardT$ minutes per passenger).

The characteristics of the line are as following. Parameter cap_k corresponds to the capacity of bus k, $dist_s$ is the distance in meters between stops s and $s-1$, $speed_{ks}$ is the operating speed in meters per minute of bus k between stops s and $s-1$ while the bus is moving, and $OD_{kss'}$ is the fraction of passengers boarding bus k at stop s whose destination is stop s' (for all $s < s'$). The headway between buses k and $k-1$ in this line must be between the interval $[minHead_k, maxHead_k]$ to be considered congruent, which is specified as an input parameter for our model.

At time t^0, instant when the holding decisions are needed, we assume that we have the following state of the transit corridor:

- d_k^0 distance between bus k and its last visited stop at time t^0. If the bus is still at a stop, then $d_k^0 = 0$.
- $s(k)$ indicates the last stop that bus k has visited at time t_0. If bus k is at stop s', then $s(k) = s' - 1$. In Figure 3, $s(2) = 3$ and $s(3) = 1$, and to simplify the notation, $s(K) = 0$, but $s(1) + 2 = S + 1$.
- c_s^0 is the number of passengers waiting at stop s at time t^0.

[1] The time interval is a parameter in our model, which could be specified by the control unit of the bus company

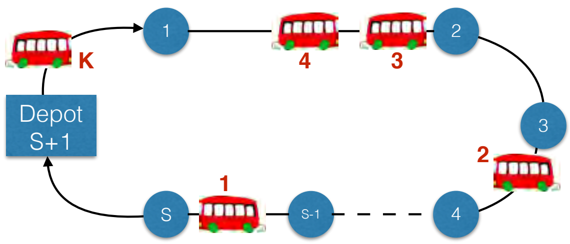


Fig. 3 Please write your figure caption here

Decision variables of our model are the holding times for each bus k at control point s, denoted by h_{ks}. There are auxiliary variables that depend on h_{ks} like the departure times of bus k at stop s that is denoted as td_{ks}. If the departure times at stop s of buses k and $k-1$ is between $[minHead_k, maxHead_k]$, then we consider that they are complying with the established headways. Nevertheless, if this difference of departure times is outside this interval, then we use the concepts of *earliness* and *tardiness* that is frequent in just-in-time scheduling theory (Rios-Solis and Sourd, 2008; Rios-Solis, 2008; Sourd and Kedad-Sidhoum, 2003; Ríos-Mercado and Ríos-Solís, 2012). The *earliness of the headway* between buses k and $k-1$ at stop s is defined as $E_{ks} = \max(minHead_k - (td_{ks} - td_{k-1s}), 0)$ which can be linearized as follows:

$$E_{ks} \geq minHead_k - (td_{ks} - td_{k-1s}),\ k = 2, \ldots, K, s = s(k)+1, \ldots, S \quad (1)$$

$$E_{ks} \geq 0,\ k = 2, \ldots, K, s = s(k)+1, \ldots, S. \quad (2)$$

While the *tardiness of the headway* is $T_{ks} = \max((td_{ks} - td_{k-1s}) - maxHead_k, 0)$ which is equivalent to

$$T_{ks} \geq (td_{ks} - td_{k-1s}) - maxHead_k,\ k = 2, \ldots, K, s = s(k)+1, \ldots, S \quad (3)$$

$$T_{ks} \geq 0,\ k = 2, \ldots, K, s = s(k)+1, \ldots, S. \quad (4)$$

Then, the objective function of BBH is the minimization of the sum of all early and tardy headways:

$$\min \sum_{k=2}^{K} \sum_{s=s(k)+1}^{S} \psi E_{ks} + \epsilon T_{ks}, \quad (5)$$

where ψ and ϵ are linear penalization for the earliness and the tardiness, respectively.

The departure times of each bus k at each stop s are defined with two different set of restrictions. The first one is the case where the bus k at time t^0 is between stops $s(k)$ and $s(k)+1$ (in Figure 3 this case would apply for bus 2 that is between stops 3 and 4). Here, the departure time of k at $s(k)+1$ is the time that needs the bus to arrive to the stop, plus the dwelling time $dwell_{ks(k)+1}$ (that will be computed later) plus the time the model decides that this bus will hold. The second case is similar but considers that the bus has not yet reached stop $s-1$. Restrictions (8) impose a limit of $maxHold$ to each holding time to guarantee a certain traveling time quality of the passengers.

$$td_{ks(k)+1} = t_0 + \frac{dist_{s(k)} - d_k^0}{speed_{ks(k)}} + dwell_{ks(k)+1} + h_{ks(k)+1}, \quad k \in K \tag{6}$$

$$td_{ks} = td_{ks-1} + \frac{dist_{s-1}}{speed_{ks-1}} + dwell_{ks} + h_{ks}, \quad k \in K, s = s(k)+2, \dots, S-1 \tag{7}$$

$$h_{ks} \leq maxHold, \quad k \in K \setminus \{1\}, s = s(k)+1, \dots, s(k-1) \tag{8}$$

From the state variables of the system, we can compute the total number of passengers that will be at stop s when bus k will reach this stop, denoted as $pass_{ks}$ in (9) and (10), as the number of passengers who are actually in the stop plus the ones that will arrive. The number of passengers that will be in bus k at stop s is equal to the passengers that want board bus k, $pass_{ks}$, minus the proportion of the passengers that left the bus before stop s (restrictions (11)). This manner, we can compute the dwell times of bus k at s (restrictions (12)). Notice that alighting and friction between the passengers that stay inside the bus could be easily included in this restriction.

$$pass_{ks} = c_s^0 + \lambda_s(td_{ks} - t_0), \quad k \in K, s = s(k)+1, \dots, s(k-1) \tag{9}$$

$$pass_{1s} = c_s^0 + \lambda_s(td_{1s} - td_{Ks}), \quad s = s(K)+1, \dots, S \tag{10}$$

$$passBus_{ks} = \min\left(\sum_{i=1}^{s-1} pass_{ki}\left(1 - \sum_{j=i+1}^{s-1} OD_{kij}\right), cap_k\right), \quad k \in K, s = s(k)+1, \dots, S \tag{11}$$

$$dwell_{ks} = passBus_{ks} boardT, \quad k \in K, s = s(k)+1, \dots, S. \tag{12}$$

The following restrictions are the different cases that need to be considered in order to avoid bus overtaking:

$$td_{ks} - td_{k-1s} \geq 0, \quad k \in K \setminus \{1\}, s = s(k-1)+1, \dots, S \tag{13}$$

$$td_{1s} - td_{Ks} \geq 0, \quad s = s(k)+1, \dots, s(1) \tag{14}$$

$$td_{k-1s} - td_{ks} \geq 0, \; k \in K \setminus \{1\}, s = s(k)+1, \dots, s(k-1). \tag{15}$$

The LP for BBP is then

$$\begin{array}{ll} \min & \sum_{k=2}^{K} \sum_{s=s(k)+1}^{S} \psi E_{ks} + \epsilon T_{ks} \\ \text{s.t.} & (1) - (4) \\ & (6) - (15) \\ & E_{ks}, T_{ks}, h_{ks} \geq 0, \quad k \in K, s \in S. \end{array}$$

Our model improves the one proposed in Delgado et al (2012) in the following aspects.

- Our objective function is linear so we obtain optimal solutions.

- We only take into account the possible holding times of a bus from its position up to the depot.
- The departure times of the buses are according to their established headway or timetable. Only perturbations that arise along the trip are taken into account.
- We may have different headways for every pair of buses. This way, recent synchronization timetables can be benefited by our approach and dealing with different planning periods (e.g., rush hour, night time) is natural.
- We do not need to call the model every time a bus arrives at a stop, only each interval of time. This fact is more realistic for a bus company. In Mexico, the companies we know that have AVL-GPS data of the buses is updated every two minutes.

4 Experimental results

The BBP LP model described in the previous section needs data to be populated. Data can be retrieved through the use of monitoring technologies like Global Positioning Systems (GPS) and Automatic Vehicle Location systems (AVL) in real-time during the execution of the bus corridor. However, to study the impact of our model under different scenarios in the traffic corridor we consider a discrete event simulation.

The single corridor is simulated using the discrete event and stochastic simulator ExtendSim AT version 9.0 (Krahl, 2009; Diamond et al, 2010). The simulator triggers an event every fixed amount of time, in which the positions of the buses and their loads, and the passengers waiting at the stops, together with their traveling destinations, are updated.

Our BBP LP model uses deterministic functions to forecast demands and travel times. Nevertheless, we use stochastic processes in the simulation to reflect a real system. We use a single corridor of 10 kilometers with 30 stops and one depot uniformly distributed like in Delgado et al (2012). There are only 30 stretches, since the last stop is merged with the depot. Travel times of the buses between each pair of stops are distributed as Lognormal with a mean of 0.77 minutes and variance of 0.4 (Hickman, 2001; Zhao et al, 2003). At each stop, passengers arrive randomly using a Poisson distribution with rate equal to one (Jolliffe and Hutchinson, 1975). The mean of the distributions are the parameters used by our model.

When passengers arrive at a bus stop, a destination is assigned to them. Passengers wait in line to board the bus in a FIFO manner. Boarding and alighting times of passengers is set to 2.5 and 1.5 seconds respectively, since all buses have two doors, one for boarding and one for alighting. If passengers cannot enter a bus because it reached its capacity, they will wait in the stop until the next bus with free space arrives. This waiting time is denoted as W_{first}. The headway time windows are set to $[minHead_k, maxHead_k] = [0.3, 0.46]$ minutes for all the buses. Remark that these time windows are easily adjustable for the cases where there are different periods along the day, and for the synchronization timetables that favor transfers.

We use a fleet of 60 buses with a maximum capacity of 100 passengers per bus. Every fixed amount of time *interval*, we determine the actions that should be followed by creating the BBP LP model in Java, and solving it with the linear package of Gurobi 5.6. The solution generated contains the holding times for all the buses for all the future stops up to the depot. If after a time *interval* a new solution is generated, then the holding times are updated using a rolling horizon scheme.

The scenarios for the simulation are divided in two parts: *time interval* scenarios and the *parameters setting* scenarios; and they are described in the next subsections.

4.1 Time interval scenarios

The aim of the time interval scenarios is to determine the optimal policy for controlling when new holding times must be computed and given to the system. In our case of study for the city of Monterrey (México), the bus company updates every two minutes the positions and all the related data of the buses in the transit corridor. Following this policy, Table 1 shows the time interval scenarios in which we test our approach.

The first column in Table 1 identifies the scenarios while the second column sets the time intervals (in minutes) in which our BBP LP model is constructed and solved to introduce the resulting holding times to the system. We vary these control values from 2 to 10 minutes. Scenario TI_0 does not have any control, and we use it as a baseline to compare the performance of our BBP LP model. The third column is an indicator if restriction (8) is applied; that is, if the holding times are bounded. For these scenarios, we set the earliness and the tardiness penalties $\psi = \epsilon = 1$. The fourth column, W_{first}, corresponds to the total average waiting time (in minutes) of a passenger to board a bus. The fifth column (Travel) represents the total average travel time of passengers in minutes, while the column Pass indicates the average number of passengers in the system during the simulation time. Last two columns indicate the normalized waiting and travel times of each passenger.

Ten simulation runs were executed for every scenario, each of them corresponding to one hour of bus operations. Each run has the same initial conditions initialized with random numbers. At the beginning of the simulation the buses are placed evenly spaced along the corridor. For each simulation run, we let the system to evolve freely for five minutes before making any holding. Indeed, five minutes is enough to observe several bus bunching situations to arise.

We observe an increase in the passenger riding time, and potentially operation costs because of the introduction of holding times in the corridor. This behavior is expected, and in concordance with other works (Furth and Muller, 2007). Nevertheless, the passenger-waiting times for the first bus are always

Table 1 *Time interval* scenarios with earliness and tardiness penalties $\psi = \epsilon = 1$.

Scen	Control (min)	$maxHold$ (min)	W_{first} (min)	Travel (min)	Pass	W_{first}/ Pass	Travel/ Pass
TI_0	✗	✗	1798.0	12035.8	1713.3	1.0	7.0
TI_1	2	✗	1115.88	17045.40	1703.1	0.66	10.01
TI_2	5	✗	1136.92	18256.20	1746.2	0.65	10.45
TI_3	7	✗	1222.66	18907.13	1705.8	0.72	11.08
TI_4	10	✗	1362.68	18652.68	1708.5	0.80	10.92
TI_5	2	0.38	1219.52	13112.15	1721.4	0.71	7.62
TI_6	5	0.38	1330.89	13171.48	1737.4	0.77	7.58
TI_7	7	0.38	1463.25	12851.01	1725.6	0.85	7.45
TI_8	10	0.38	1424.52	12450.34	1697.7	0.84	7.33

reduced, which in fact it is what we wanted to show in first place. Indeed, by controlling the headway we can also control the passenger-waiting times, without the need of using a quadratic objective function in the model.

We can also observe that the best passenger-waiting times are for the cases where the holding controls are applied every 2 to 5 minutes, and without the bounds on the holding times. However, the bounds on the holding times induce a reduction on the travel times, which is an important asset. Figure 4 shows the differences in performance when the control (8) ($maxHold$) is applied. The left hand side of the figure shows the percentage of increase on the passenger-waiting times when bounds are applied, while its right hand side illustrates the percentage of increase on the travel times when they are not applied. As mentioned, we observe that even if there is an increase on the passenger-waiting times when bounds are applied, the benefit on the travel times is considerable.

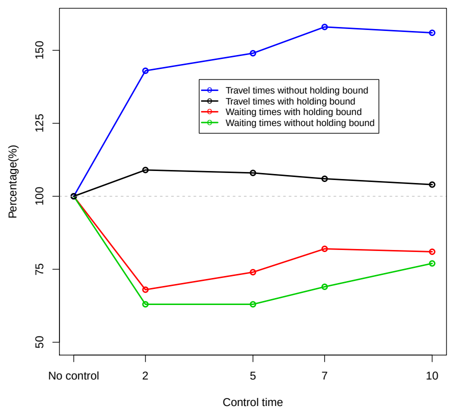


Fig. 4 Decrease on the waiting times and increase of the travel times for the *time interval* scenarios with earliness and tardiness penalties $\psi = \epsilon = 1$ and applying bound to holding time.

For a bus company, the less the traffic controller has to give holding orders to the system (i.e., bus drivers), the better. Therefore, from Table 1 and Figure 4, we conclude that the best policy is to consider bounds on the holding times, and apply the controls to the system every 5 minutes, like in the TI_6 scenarios.

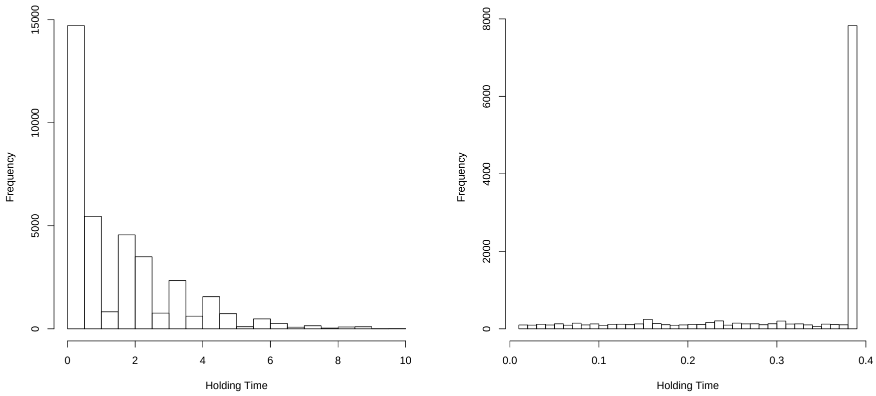
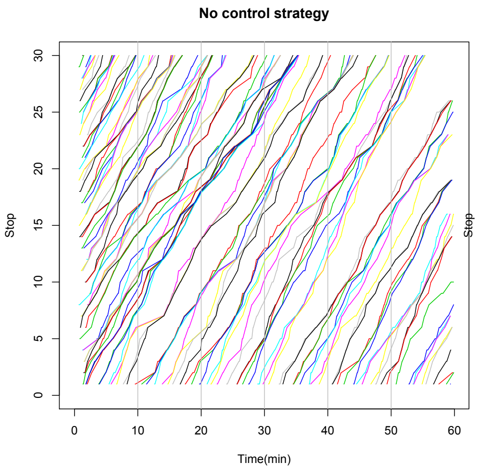


Fig. 5 Holding times histogram without bounds (left histogram) and with bounds (right histogram) for the *time interval* scenarios with earliness and tardiness penalties $\psi = \epsilon = 1$, and a control of 5 minutes.

In Figure 5, we show two histograms of the length of the holding times (x-axis in minutes) for the time interval scenarios with earliness and tardiness penalties $\psi = \epsilon = 1$, and a control of 5 minutes with and without bounds. On the y-axis, we have the frequency the BBP LP model is called for all the simulations of class TI_6. Notice that not all of the holding times are applied, since the rolling horizon may modify several of them. The case when there are limits on the holding times shows that the model either chooses to apply the holding times close to these limits, or not to apply them at all. This is an implicit benefit for the users, and for the traffic controller.

The aim of the BBP model is to reduce bus bunching by maintaining congruent headways. To graphically show that this behavior is being improved by our model, we present Figures 6-9 for the scenarios with bounds on the holding times. The x-axes in these graphs correspond to time (in minutes), while the y-axes represent stops. Each line in these graphs represents a bus that departs from the depot and cruises all the bus stops. Recall from Section 1 (see Figure 1) that in the ideal case, we would have *parallel* lines. Figure 6 displays the case without control. Here the bus bunching problem is notorious, since there are white gaps between the lines. Figures 7, 8, 9, have time interval controls of 2, 5, and 7 minutes respectively. We can observe that with 2 and 5 minutes controls the BBP is reduced, while for control intervals of 7 minutes the BBP appears more.



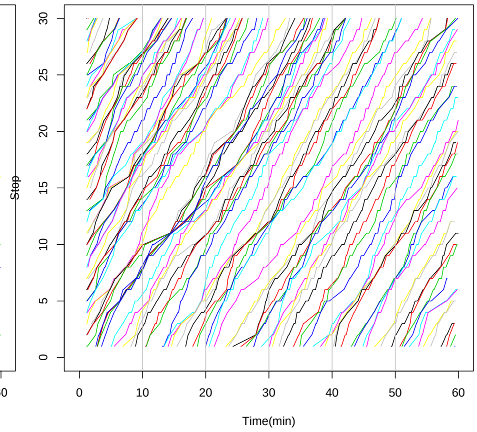
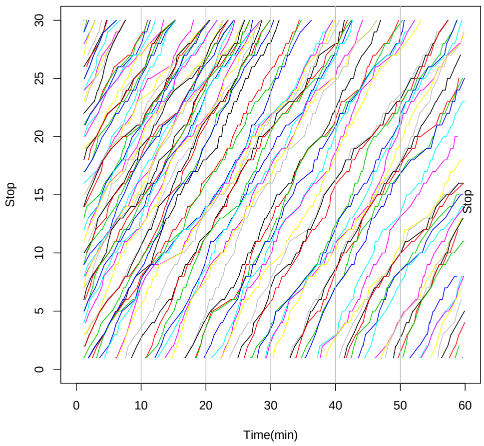


Fig. 6 Bus transit behavior without control.

Fig. 7 Transit with control every 2 min.



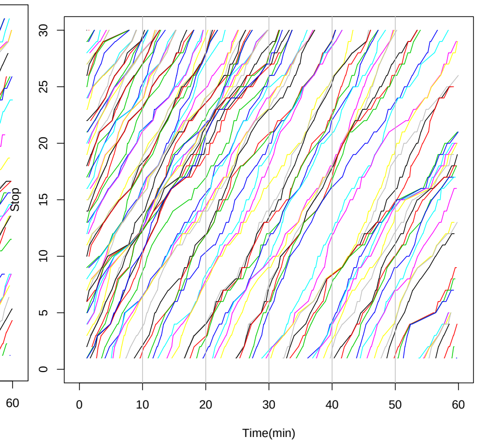


Fig. 8 Transit with control every 5 min.

Fig. 9 Transit with control every 7 min.

Figure 10 shows two histograms that have in their x-axes the round time of a bus trip. An aspect that we noticed from Table 1 is that the travel times increase with the BBP model. This is obvious because the BBP model introduces holding times for the buses in the corridor. Nevertheless, Figure 10 shows that the standard deviation when BBP is applied every five minutes (right histogram) is reduced with respect to the case where no controls are used (left hand side histogram).

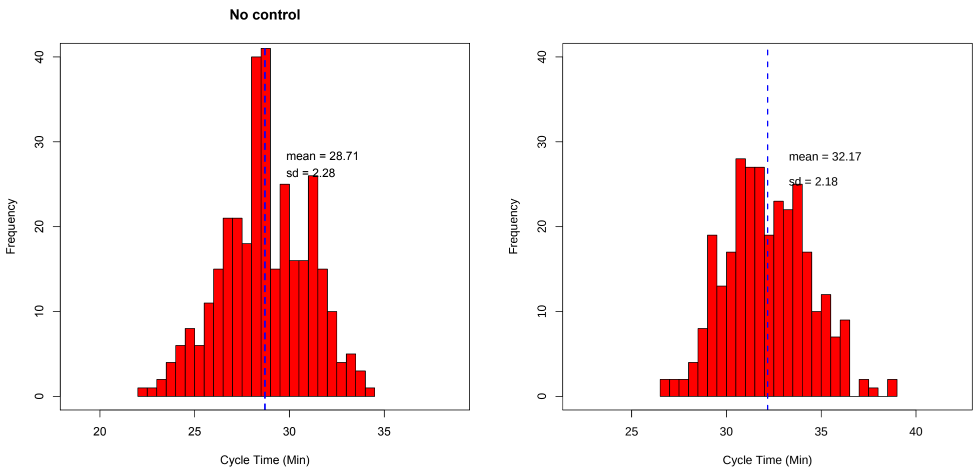


Fig. 10 Histogram of travel cycle without control (left) and with control interval of 5 minutes (right).

4.2 Parameter setting scenarios

Our next set of experiments modify the earliness ψ and tardiness *epsilon* parameters of the BBP LP objective function to observe the impact they have in the passenger-waiting times and travel time. We can see this set of experiments in Table 2. The first column in the table identifies the scenarios. Ten simulation runs were considered per scenario. The second column represents the values of the earliness parameter, while the third one corresponds to the tardiness one. The column "Board" denotes the average time (in seconds) a passenger takes to board a bus, while $maxHold$ stands for the time (in minutes) that the holding times are bounded. This table shows the percentage of reduction in passenger-waiting times (W_{first}), and the percentage of increase on the travel time (Travel). Finally, the last column represents the addition of the last two values. Indeed, if there is a reduction on this last column, the percentage would be negative.

An interesting observation from the results is that if we reduce the earliness parameter, we obtain the best results with respect to the passenger-waiting and travel times. Moreover, the BBP LP model yields better results when the holding times are limited by 0.19 minutes, which is also a quality asset for the user.

An statistical analysis confirms the observations from Table 2. The most influential parameters are the earliness penalty and the $maxHold$ limit. In Table 3, we show a linear regression of the parameters studied in this section. The first column is the parameter, the second one corresponds to the "Estimate", the third is the standard error, the fourth stands for to the t value, and the fifth one is the significance.

Table 2 Improve of the behavior of waiting time and travel time managing parameters

Scen	ψ	ϵ	Board (sec)	$maxHold$ (min)	W_{first} % reduction	Travel % increase	W_{first} +Travel % increase
P_1	0	1	1.25	0.19	19%	-2%	-4%
P_2	0	1	1.25	0.38	22%	1%	-2%
P_3	0	1	2.5	0.19	22%	-4%	-6%
P_4	0	1	2.5	0.38	25%	-1%	-4%
P_5	0.5	1	1.25	0.19	34%	10%	4%
P_6	0.5	1	1.25	0.38	55%	39%	27%
P_7	0.5	1	2.5	0.19	37%	11%	5%
P_8	0.5	1	2.5	0.38	56%	41%	28%
P_9	1	0	1.25	0.19	40%	11%	5%
P_{10}	1	0	1.25	0.38	59%	42%	29%
P_{11}	1	0	2.5	0.19	44%	12%	5%
P_{12}	1	0	2.5	0.38	63%	48%	34%
P_{13}	1	0.5	1.25	0.19	36%	10%	4%
P_{14}	1	0.5	1.25	0.38	54%	41%	28%
P_{15}	1	0.5	2.5	0.19	39%	11%	5%
P_{16}	1	0.5	2.5	0.38	57%	40%	27%
P_{17}	1	1	1.25	0.19	39%	11%	5%
P_{18}	1	1	1.25	0.38	48%	27%	17%
P_{19}	1	1	2.5	0.19	39%	56%	43%
P_{20}	1	1	2.5	0.38	57%	50%	36%

	Estimate	Std. Error	t value	Pr(> $\|t\|$)
(Intercept)	1.0314	0.0928	11.11	0.0000
ψ	-0.2234	0.0489	-4.57	0.0004
ϵ	0.0273	0.0489	0.56	0.5848
Board	-0.0845	0.0646	-1.31	0.2106
$maxHold$	-0.2956	0.0646	-4.57	0.0004

Table 3 Linear regression on the main parameters of the BBP model

5 Concluding remarks

In this paper, we presented a methodology based on interleaving planning and execution to maintain congruent headways in a bus corridor with the aim of solving one of the most annoying problems in public transit networks, the Bus Bunching Problem (BBP).

During the planning phase of our approach, a linear programming optimization model is built and solved to determine the optimal holding times of the buses at the stops to avoid bus bunching. Our model requires real-time data of the state of the system to operate. Such data is provided by the execution phase of our approach, which in our case is supported via simulation. The simulation phase of the system provides data related to positions of the buses, number of passengers in the buses, current bus capacities, and number of passengers waiting at the stops to build our model.

One of main the advantages of considering simulation in our methodology is the evaluation of multiple parameters to assess the impact of them in our BBP linear programming model. Therefore, we presented a comprehensive evaluation of such parameters, and found that applying holding controls just every 5 minutes, and bounds on the holding times reduce not only bus bunching frequency but also passenger-waiting times.

We also discussed that most of the works in the literature minimize passenger-waiting times, or the variance in the departure times of the buses using quadratic optimization functions, which are more complex to solve. Instead, the linear programming model of our approach makes it suitable for returning optimal solutions efficiently and for interleaving planning and execution phases in real-time scenarios.

Although, we observe an increase in the travel time of passengers given the introduction of holding times for the buses in the corridor, our approach performs better (i.e., less passenger-waiting time and acceptable travel time) than no introducing any control into the system. Part of our future work will consider the introduction of other actions into our models to reduce the travel time of the passengers in the corridor and lower operational costs. Particularly, we believe that the introduction of bus overtaking actions (i.e., skipping stops) will balance the total time a passenger spends into the system.

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