APPLICATION OF A DYNAMIC MODEL TO THE SIMULATION OF THE COMPOSTING PROCESS

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SUMMARY: In this work, it is presented a dynamic model in order to simulate of the biological composting process, applied to the organic fraction of the municipal solid waste. The model was applied to small scale composting units (in-vessel type), processing household wastes composed of food wastes, yard wastes and paper wastes. The model is based in a continuous plug flow reactor for the solids and in a continuous stirred tank reactor for the gas phase. The output features of the model includes the instantaneous oxygen consumption, the solids output and the solids temperature and moisture profiles. The results show that with proper system design and operation, home composting seems to ensure effective pathogen destruction, considering the modelled time/temperature profiles.

1. INTRODUCTION

The composting process applied to the treatment of biodegradable waste materials is one of the most adequate ways to enhance the environmental performance of the municipal solid waste (MSW) management systems. In the european context where environmental policies tend to encourage solid waste minimization and recycling, considering for example the targets established by the Directive 99/31/EC (European Commission, 1999) for biowaste landfill disposal, home composting is an effective method to decrease the quantity of MSW to be disposed. Home composting can be used for the conversion of the organic fraction of household wastes to a stable humus-like material (the compost).

The interest of composting is the simplicity of the process implementation, the low cost of the equipment required and of the operation, and the high degree of sanitization achieved for the waste. Nevertheless, the compost quality depends primarily of the non-contaminated origin of organic wastes to be composted and of properly process design and operation. A great advantage of this process of waste treatment is the easily of scaling up from the household level to the multi-municipal level. In fact, this process is useful for household food and garden wastes, as well as biodegradable wastes derived from agriculture and industry of food products.

The composting is an aerobic biological process during which the biodegradable organic substrates are decomposed by microorganisms (bacteria and fungi) to give mainly carbon dioxide and water and a solid phase (the compost), with heat release. This process involves diverse linked phenomena of physical, chemical and biological nature, and is influenced by
operational factors, constituting a heterogeneous system of great complexity. Hence, to better understanding the mechanisms involved in the composting process and with the purpose of properly sizing and optimizing reactive systems, some mathematical models have been developed according to diverse approaches and degree of complexity (Gomes, 2001).

In this work a composting dynamic model is presented, which development was based on a macroscopic viewpoint of the process, on the basis of physical and chemical considerations, in order to evaluate the performance of small scale composting units applied to household wastes.

To implement the model, it is necessary to specify the excess air ratio and the composition and feed rate of the mixture of solid wastes to be treated. Then, it is established mass and energy balances to the composting process based on a daily solid and air feeding rates.

The proposed model makes possible to simulate several outputs, including the temperature profile of the composting materials, the evolution of solids moisture, the aeration rate, the accumulated mass of wastes until steady state is achieved (which is the total mass of wastes in composting at steady state) and other features of the system in steady state operation. The interest of this model includes the evaluation of the process conditions (temperature and time or space) which lead to a suitable degree of pathogen control in compost, and moisture of the solids.

The dynamic simulation of the system was performed using the Extend™ software, that offers an intuitive graphical visualization of the structure of the model and great flexibility in what concerns the establishment of sensitivity analyses for different model parameters and variables.

2. THE DYNAMICS OF THE COMPOSTING PROCESS

As an alternative to static models, which are used to study the behaviour of systems in steady-state, dynamic models help us to understand how a given system evolves over time (for example, to study the changes experimented by a system between two equilibrium conditions).

In fact, time is an important factor to be considered when analysing the composting process, since almost all process variables present an evolution over time. Considering these variables inter-relations on time dependence, to improve the understanding of the dynamics of the composting process a generalised causal loop diagram is presented in Figure 1. Some cause-and-effect relationships showed in Figure 1 are, for example: (i) higher substrate biodegradability means higher rate of substrate decomposition, higher rates of released heat and increased

![Figure 1. Generalised causal-loop diagram of the composting process.](image-url)
substrate temperature; but, (ii) higher temperature leads to increase the rate of substrate decomposition and to increased rate of water evaporation, and so on.

Proper system design, optimization and process control, involves the understanding of these relationships of the composting process, since they explain its dynamics. So these kind of causal-loop diagrams are very useful in the field of control engineering.

3. MODELING THE COMPOSTING PROCESS

3.1 Conceptual physical model

The model presented for the composting process is based on a steady-state heterogeneous bioreactor, completely stirred for the gas phase and plug-flow for the solids, with continuous feeding of air and of solid wastes and continuous discharge of gases and solid product (compost). A one step first order kinetics for the aerobic decomposition of solids was considered, and stoichiometric relationships were established.

It was assumed that process control involves mainly the adjustment of the airflow rate through an aeration system that distributes air within the base of the composting materials (forced ventilation system). Air is supplied to the system in excess of the stoichiometric requirements, in order to satisfy the stoichiometric oxygen demand and to control the moisture and temperature of the solid wastes. In fact, it is usual to operate composting systems with excess air, since the aeration rate required for heat and water removal is greater than stoichiometric requirements.

In the modelling approach there isn’t mixing of the solid materials or water addition, although it is recognised the importance of these process variables in the management of the system. It wasn’t considered also the removal of water from the composting system through the formation of leachates.

At steady state operation, the bioreactor receives daily amounts of wastes in one side and the compost is discharged daily on the other side. The successive loads of wastes remain segregated along the bioreactor, and it was considered a processing time period of 30 days.

3.2 Composting materials

In Portugal, MSW generation trends show a growth of approximately 30% since 1995 to 2005, and it was estimated in approximately 4.5 Mton.year\(^{-1}\) in 2005 (www.inresiduos.pt). Considering the disposal of MSW in 2005, near 63% was landfilled, 21% was treated in incineration plants, 9% was separately collected and 7% was treated in composting plants (www.inresiduos.pt). Nevertheless, the physical composition of MSW shows that about 30-40% (w/w) of the actual MSW generation is suitable for composting (Magrinho et al., 2006).

Although the composition of MSW generated within a municipality could vary widely, a review of some MSW physical characterization campaigns in Portugal shows that the composition of the mixture of wastes suitable for home composting has approximately the pattern presented in Table 1. Proper composting system design and optimization requires an accurately estimation of the feeding wastes macroscopic composition, and corresponding proximate and elemental analysis, biodegradability and kinetic parameters. Typical values of the macroscopic components ultimate analysis are present in Table 2 and it was considered that the compost ultimate composition was of the same of the initial mixture, in ash-free dry basis.

The present dynamic model was applied to simulate the composting of selectively separated biowastes generated in a house with 5 persons, in a small scale composting system, according to the mixture specifications of Table 1 and Table 2. The composting system receives \(m_{R0}=1,074\) kg·day\(^{-1}\) (db – dry basis) of wastes (or 2,776 kg·day\(^{-1}\) wb –wet basis), composed of \(m_{FW0}=0,601\)
kg·day\(^{-1}\) (db) of food wastes, \(m_{YW_0}=0,151\) kg·day\(^{-1}\) (db) of yard wastes and \(m_{PW_0}=0,318\) kg·day\(^{-1}\) (db) of organic contaminated paper wastes.

Table 1: Initial composition, biodegradability and kinetic parameters of the mixtures of wastes \(^1\) Lipor, 2000 and www.ersuc.pt; \(^2\) Tchobanoglous et al., 1993; \(^3\) Keener et al., 1993).

<table>
<thead>
<tr>
<th>Waste type</th>
<th>Mass fraction (^1) (wb)</th>
<th>Per capita generation (^1) (kg·dia(^{-1}) wb)</th>
<th>Moisture (^2) (kgH(_2)O·kg(^{-1}) wb)</th>
<th>Biodegradability (^3) (db)</th>
<th>First order kinetic constant (at 55 °C) (^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food wastes (FW)</td>
<td>0,728</td>
<td>0,401</td>
<td>0,7</td>
<td>0,55</td>
<td>0,25</td>
</tr>
<tr>
<td>Yard wastes (YW)</td>
<td>0,137</td>
<td>0,075</td>
<td>0,6</td>
<td>0,51</td>
<td>0,18</td>
</tr>
<tr>
<td>Paper wastes (PW)</td>
<td>0,135</td>
<td>0,075</td>
<td>0,15</td>
<td>0,46</td>
<td>0,069</td>
</tr>
</tbody>
</table>

Table 2: Chemical characterization of the wastes (db) (Tchobanoglous et al., 1993).

<table>
<thead>
<tr>
<th>Waste type</th>
<th>Ash</th>
<th>w(_C)</th>
<th>w(_H)</th>
<th>w(_O)</th>
<th>w(_N)</th>
<th>w(_S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food wastes (FW)</td>
<td>0,050</td>
<td>0,480</td>
<td>0,064</td>
<td>0,376</td>
<td>0,026</td>
<td>0,004</td>
</tr>
<tr>
<td>Yard wastes (YW)</td>
<td>0,045</td>
<td>0,478</td>
<td>0,060</td>
<td>0,380</td>
<td>0,034</td>
<td>0,003</td>
</tr>
<tr>
<td>Contaminated paper wastes (PW)</td>
<td>0,060</td>
<td>0,435</td>
<td>0,060</td>
<td>0,440</td>
<td>0,003</td>
<td>0,002</td>
</tr>
<tr>
<td>Mixture of wastes (R)</td>
<td>0,052</td>
<td>0,466</td>
<td>0,062</td>
<td>0,396</td>
<td>0,020</td>
<td>0,003</td>
</tr>
</tbody>
</table>

3.3 General composting model description

3.3.1 Process kinetics and system design

Composting kinetics describes the dependence of the wastes decomposition rate on environmental factors, such as temperature, oxygen availability, moisture, substrate particle size, porosity, etc. (Hamelers, 2004). In the composting model presented, the rate of substrate decomposition was described by the first order model of Keener et al. (1993) by

\[
(-R_g) = -\frac{dm_R}{dt} = k_{1.9} \cdot \left(\frac{T-15}{5}\right) \cdot m_R \cdot \left(m_c - m_{W_0}\right)
\]

where the kinetic parameters depend on the composting materials and temperature, according with Schulze (1962).

This model was applied to each component of the mixture of wastes (see Table 1) and its output is the actual mass of the composting materials \((m_{FW}, m_{YW}, \text{ and } m_{PW})\). Since it is modeled the evolution of the mass of the mixture of wastes \((m_R)\) throughout the processing period \((t_i = 30\) days\), it is possible to predict the mass of waste gasified per day \((m_{RG})\) when the system is operating in steady-state:

\[
m_{RG} = \sum_{i=0}^{t_i-1} \left[ m_R(i) - m_R(i+1) \right]
\]

If the temperature of the overall process will be constant along time, it can be shown that the last equation gives:

\[
m_{RG} = (m_{W_0} - m_{W_0}) \cdot (e^{-t_i} - 1) \cdot \left(\frac{1-e^{t_i}}{1-e^{-t_i}}\right)
\]

The compost discharge rate at steady-state can now be evaluated by performing a mass balance to the composting system in steady-state operation, since both the feed rate of wastes
(\(m_{R0}=1.074\) kg-day\(^{-1}\) (db)) and the waste gasification rate is known. As Eq.1 describes the evolution of the mass of composting materials, it is also possible to calculate the mass of solids accumulated in the system during the processing time, which leads to evaluate the total mass of wastes in composting at steady-state. By modelling the evolution of the bulk density and the moisture of the solids, then the required reactor volume can be evaluated:

\[
V_C = \int_0^t m_R(t) \frac{1}{1-W_{SR}(t)} \frac{1}{\rho_R(t)} \, dt
\]

### 3.3.2 Oxygen demand

Oxygen inflow was modeled by considering that ambient air (with 0.21 v/v of oxygen) is supplied by an aeration system with the ability of control the aeration rate, since this is one of the key points of the composting process control. The instantaneous stoichiometric oxygen demand is determined from the rate of materials decomposition (see Eq.1) and represents the rate at which the oxygen must be supplied over the processing time period to meet the stoichiometric demand:

\[
(-R_{O}) = \left(-\frac{dm_{O}}{dt}\right) = M_{SR} \left(-\frac{dm_{R}}{dt}\right)
\]

where \(M_{SR}\) is the stoichiometric oxygen demand of wastes, determined from the chemical composition of the wastes and the compost (Table 2), presented by Matos and Pereira (2003):

\[
M_{SR} = \sum_j \left(\frac{m_{j} \cdot w_{j}}{M_j}\right) \frac{w_{O}}{O_2} - \left(\frac{m_{j} \cdot w'_{j}}{M_j}\right) \frac{w'_{O}}{O_2}
\]

If \(z\) is the excess of oxygen then the actual oxygen demand is \(M_{A} = (z + 1) \cdot M_{SR}\) and the rate of actual oxygen uptake by the composting process is:

\[
(-R_{O})_A = M_{A} \left(-\frac{dm_{R}}{dt}\right)
\]

### 3.3.3 Energy and water balances

In composting, energy is generated during the biodegradable organic solids oxidation in the form of heat release (enthalpy of reaction). Also, energy enters the composting system through the ambient air (sensible and latent heat), and exits the system through the flue gas (sensible and latent heat and the heat of reaction associated to the presence reduced gaseous species, namely ammonia (NH\(_3\)) and hydrogen sulphide (H\(_2\)S)). The enthalpy accumulated (\(E_{Ac}\)) in the composting materials along the processing time can be evaluated through,

\[
\frac{dE_{Ac}}{dt} = N_j - N_k + N_{O} - N_u
\]

where \(N_j\) is input rate of energy associated to the inflow rate of ambient air:

\[
N_j = (-R_{R}) \cdot M_A \left[\tau_{P_{O2}} + \gamma_{N_2}\tau_{P_{N_2}} + \gamma_{air} \cdot \tau_{W_{VA}} \cdot \tau_{P_{V}} \right] \left(T_j - T^0\right) + (-R_{R}) \cdot M_A \cdot \gamma_{air} \cdot \gamma_{H_2O} \cdot W_{VA} \cdot M_{H_2O} \cdot h_{WV}
\]

\(N_E\) is the output rate of energy associated to the composting flue gas rate:
with  

\[ n_{O_2} = z \cdot M_{SR} \cdot n_{N_2} = M_A \cdot \gamma_{N_2} \cdot n_{H_2O} = z \cdot M_{SR} \cdot \frac{1}{y_{O_2}} \cdot W_{VE} \quad \text{and} \quad W_{VE} = \frac{p_w}{p_1 - p_a} \]

\[ N_U = U \cdot A_R \cdot (T_R - T_S) \]  

\[ T_R = \frac{E_{AC}}{m_R \cdot \left( \frac{\varepsilon}{p_R} + H_S \cdot \frac{\varepsilon}{p_W} \right)} + T^0 \]  

\[ \frac{dm_{Ac}}{dt} = (-R_R) \cdot M_A \cdot \varepsilon \cdot W_{VE} \cdot M_{H_2O} + (-R_R) \cdot n_{H_2O} \cdot M_{H_2O} - (-R_R) \cdot n_{H_2O} \cdot M_{H_2O} \]  

The main feature of the composting model, implemented with Extend™ software, is the simultaneous numerical resolution of the differential equations Eq.1, Eq.8 and Eq.14. The simulation of the solids dry mass, energy and water stocks, for each load of wastes, is the central objective of the composting process modelling, since all other process variables are readily evaluated through simple relationships with these stocks. The Extend™ software is an efficient programming tool to implement dynamic models, specially in what concerns the sensitivity analyses to diverse model parameters.
Table 3 presents the composting process model constants and variables, and their corresponding definitions, values and units.

Table 3 – Constants and variables used in the composting process model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{ps}$</td>
<td>1.45</td>
<td>kJ·kg$^{-1}$·K$^{-1}$</td>
<td>Average specific heat of the mixture of wastes R</td>
</tr>
<tr>
<td>$LHV_{R0}$</td>
<td>16560</td>
<td>kJ·kg$^{-1}$</td>
<td>Lower heat value of wastes (based on Dulong formula)</td>
</tr>
<tr>
<td>$LHV_{Re}$</td>
<td>15550</td>
<td>kJ·kg$^{-1}$</td>
<td>Lower heat value of compost (based on Dulong formula)</td>
</tr>
<tr>
<td>$M_{SR}$</td>
<td>1.89·10$^{-2}$</td>
<td>kmolO$_2$·kg$^{-1}$</td>
<td>Stoichiometric oxygen demand</td>
</tr>
<tr>
<td>$n_{CO2}$</td>
<td>1.75·10$^{-2}$</td>
<td>kmolCO$_2$·kg$^{-1}$</td>
<td>Generated mol of CO$_2$ per kg of waste processed</td>
</tr>
<tr>
<td>$n_{NH3}$</td>
<td>6.52·10$^{-3}$</td>
<td>kmolNH$_3$·kg$^{-1}$</td>
<td>Generated mol of NH$_3$ per kg of waste processed</td>
</tr>
<tr>
<td>$n_{H2S}$</td>
<td>4.59·10$^{-3}$</td>
<td>kmolH$_2$S·kg$^{-1}$</td>
<td>Generated mol of H$_2$S per kg of waste processed</td>
</tr>
<tr>
<td>$n_{H2O}$</td>
<td>1.30·10$^{-2}$</td>
<td>kmolH$_2$O·kg$^{-1}$</td>
<td>Generated mol of H$_2$O per kg of waste processed</td>
</tr>
<tr>
<td>$U$</td>
<td>43</td>
<td>kJ·day$^{-1}$·m$^{-2}$·K$^{-1}$</td>
<td>Overall heat transfer coefficient (based in conduction)</td>
</tr>
<tr>
<td>$e_g$</td>
<td>0.48</td>
<td>kg$<em>{Re}$·kg$</em>{R0}^{-1}$</td>
<td>Global yield of the composting process (db) ($= m_{Re}/m_{R0}$)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.61</td>
<td>kg$<em>{air}$·kg$</em>{H2O}^{-1}$</td>
<td>$M_{by~air}/M_{H2O}$</td>
</tr>
<tr>
<td>$\gamma_{N2}$</td>
<td>3.76</td>
<td>kmolN$_2$·kmol$^{-1}$O$_2$</td>
<td>$y_{N2}/y_{O2}$ (in dry air)</td>
</tr>
<tr>
<td>$\gamma_{air}$</td>
<td>4.76</td>
<td>kmol$_{dry~air}$·kmol$^{-1}$O$_2$</td>
<td>$1/y_{O2}$ (in dry air)</td>
</tr>
<tr>
<td>$W_{VA}$</td>
<td>0.012</td>
<td>kg H$_2$O·kg$^{-1}$ dry air</td>
<td>At $T_i$ and 60% relative humidity</td>
</tr>
</tbody>
</table>

4. RESULTS

4.1 Evolution of the mass of materials in composting

Using Eq.1 and the data presented in Table 1 it is possible to simulate the evolution of the mass of each component of the mixture of wastes, which total initial mass was 1,074 kg (db). In Figure 2 it is possible to see that, although food wastes were the main component in the initial mixture, its higher biodegradability and kinetics makes its mass fraction to decrease rapidly in the first 5 days (approximately 40 % of mass reduction). Maximum rates of composting occur at around 60 hours of treatment.

Figure 2: Simulated evolution of the mass of composting materials (legend according to Table 2).

Figure 3: Accumulated mass of composting materials and compost discharge rate in steady-state operation.
4.2 Evolution of oxygen consumption

The instantaneous stoichiometric oxygen requirements of each component of the mixture of wastes are presented in Figure 4. The rate of oxygen consumption peaks at approximately 2.5 days of composting with a total rate of 2.6 molO₂·day⁻¹ (≈ 0.4 kg air·day⁻¹). Food wastes are responsible for the main consumption of oxygen. It should be noted that maximum oxygen uptake rates for yard wastes and paper wastes are delayed from the peak of food wastes, which is a consequence of the corresponding values of first order kinetic constants.

This stoichiometric oxygen demand of the composting system points out the importance of the ventilation apparatus in composting operations, in what concerns to its ability to control the aeration rate (over time or over the bioreactor space). In the case of home composting, the predicted total rate of stoichiometric oxygen demand ranges from 0.1 molO₂·dia⁻¹ up to 2.67 molO₂·dia⁻¹ (Figure 4).

Concerning the operation of the composting system in steady state, Figure 5 shows that daily stoichiometric requirements of oxygen is approximately 0.27 kgO₂·dia⁻¹. If we consider the actual rate of oxygen supply (see Eq.7) and an excess of oxygen of 300% the stoichiometric demand, the ventilation system should be able to supply 1.1 kgO₂·dia⁻¹ (≈ 4.7 kg air·day⁻¹) to the composting system at steady-state operation.

4.3 Energy, temperature and moisture of the composting materials

The initial temperature of wastes (1,074 kg·dia⁻¹ (db) initial mass), daily introduced in the composting system, was considered to be the same of the ambient air temperature (25 ºC). The excess air selection strategy was based on the subsequent modelled time/temperature and time/moisture profiles of the solids, in order to maintain the temperature and moisture at optimal levels (minimize the pathogen risk, prevent heat inactivation, maintain wet substrates and reduce leachates). The data used in the energy and water balances is summarized in Table 3.

Figure 6 shows a plot of the predicted thermal energy and corresponding temperature of the solids during the processing period (see Eq.8 and Eq.13). The energy content of the solids rises rapidly just the process starts due to heat of reaction release. During the first 50 hours of treatment, this rate of energy generation is greater than the energy output of the system, through the energy content of the exhaust gas and heat transfer to the surrounding environment. After this
initial period, the energy output is always greater than the energy input, so the thermal energy of the materials peaks 58 hours after the process start and then decreases rapidly.

Simulated temperature curve shows a maximum of 62 °C after 3 days of treatment (Figure 6). Temperature is expected to be above 60 °C during 27 hours and above 55 °C during 54 hours, which seems to ensure a reasonable degree of pathogen destruction, considering for example the literature review presented by Haug (1993) about this subject.

The solids moisture/time profile is presented in Figure 7. For the modelled conditions, the solids moisture increases over time, after an initial fluctuation during the period of high-rates of substrate decomposition. The sensitivity analyses performed shows that solids moisture always growth after the initial high-rate stage of composting, due to substrate temperature decrease thereafter and hence the decrease of water vapour transport capacity of the exhaust gas. So, the model predicts that formation of leachates is very likely to occur if the solids moisture increases over its holding water capacity.

![Figure 6: Evolution of materials (dry substrate plus moisture) thermal energy and corresponding temperature profile.](image1)

![Figure 7: Predicted evolution of the moisture of the composting materials.](image2)

5. Conclusions

The composting model shows that with proper system design and aeration control it seems that home composting effectively ensures proper compost sanitation. In fact, the solids time/temperature profile is very sensitive to the excess air ratio and the overall heat transfer coefficient. The model also shows that to control the compost moisture it is particularly important to control the solids moisture during the first days of composting, since the system trend is to accumulate water after the initial high-rate stage of composting.

REFERENCES


**NOMENCLATURE**

* \( \bar{c}_p \): Average specific heat of *i* compound, with *i* = N₂, O₂, CO₂, H₂O, NH₃, H₂S, R [kJ-k mole⁻¹ K⁻¹]

* \( E_{ac} \): Energy accumulated in each load of waste [kJ]

* \( H_s, W_{wR} \): Moisture content based on dry weigh and wet weigh of materials, respectively [kg H₂O⋅kg⁻¹]

* \( h_v \): Latent heat of vaporization of water at \( T_0 \) [kJ kg⁻¹]

* \( k_{55} \): Reaction rate constant of the composting process at 55 °C (see Table 1) [dia⁻¹]

* \( m_{Aw} \): Water mass accumulated in each load of waste [kg H₂O]

* \( m_{g} \): Actual dry mass of composting materials at the instant \( t = (m_{FW} + m_{YW} + m_{PW}) \) [kg]

* \( m_{g0} \): Initial mass of dry wastes (also feed rate of wastes) \( = (m_{FW0} + m_{YW0} + m_{PW0}) \) [kg, kgₘₐₜₜ⁻¹]

* \( m_{gr} \): Non biodegradable dry mass, for a time period of the order of \( t_t \) (see Table 1) [kg]

* \( m_{gR} \): Gasified dry mass waste for system operating in steady-state [kg]

* \( m_{o2} \): Instantaneous stoichiometric oxygen demand [k mol₂⁻¹]

* \( m_{s_j} \): Oxidation stoichiometric coefficient of *j* element, with *j* = C, N, S, H₂ [k mol₂⁻¹ k mol⁻¹]

* \( M_{kg}, M_{A} \): Stoichiometric oxygen demand and actual oxygen demand [k mol₂⁻¹ k g⁻¹]

* \( M_{j}, M_{i} \): Molecular weight of *j* element and *i* compound, respectively, with *j* = C, N, S, H₂ and *i* = N₂, O₂, CO₂, H₂O, NH₃, H₂S [kg k mol⁻¹]

* \( n_{j,R0} \): Water quantity in flue gas per mass of degraded wastes [k mol₁₂₀⁻¹ k g⁻¹]

* \( N_{I} \): Input rate of energy associated to the inflow of ambient air [kJ day⁻¹]

* \( N_{E} \): Output rate of energy associated to the composting flue gas [kJ day⁻¹]

* \( N_{O} \): Rate of energy production by the biological oxidation [kJ day⁻¹]

* \( N_{U} \): Rate of energy transfer between the composting materials and the surroundings [kJ day⁻¹]

* \( LHV_{i} \): Lower heating value of *i*, with *i* = R₀, Rₑ (dry compost), H₂S and NH₃ [kJ kg⁻¹]

* \( \left(- R_{g}\right), \left(- R_{o2}\right) \): Rate of substrate decomposition and rate of oxygen uptake [kgₘₐₜₜ⁻¹, k mol₂⁻¹ day⁻¹]

* \( t_{t} \): Processing time period [day]

* \( T_{0} \): Reference temperature [K]

* \( T_{R}, T_{j} \): Composting materials temperature and ambient air temperature (= 298 K) [K or °C]

* \( V_{c} \): Required volume of the composting system [m³]

* \( w_{j, w_{j}} \): Mass fraction of *j* element in dry waste (R₀) and dry compost (Re), respectively, with *j* = C, H, O, N, S (see Table 2) [kg j⋅kg⁻¹]

* \( \rho_{v} \): Saturation vapor pressure at \( T_{R} \) [Pa]

* \( \rho_{R} \): Bulk density of the composting materials (wb) [kg R⋅m⁻³]