IOM 674: Queueing and Stochastic Networks
Professor Amy Ward

Homework #4

Single Station Queues and Generalized Jackson Networks: RBM approximations

Due on Friday, April 15

1. Question 6 in Chen and Yao, page 117

2. Consider a GI/GI/1 queue in which arrivals are Poisson at rate ρ, and service times follows a gamma distribution with shape parameter k and scale parameter θ. Then, the pdf of the service time distribution is

$$f(x) = x^{k-1}\frac{\exp(-x/\theta)}{\Gamma(k)\theta^k},$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the Gamma function.

   (i) What is the appropriate RBM approximation for the number-in-system process, and the resulting mean number-in-system approximation (in terms of the given parameters)?

   (ii) Simulate the GI/GI/1 queue using ExtendSim when $\rho = 0.9$ and also when $\rho = 0.95$ in both the cases (A) that $k = 5$ and $\theta = 1/5$ and (B) that $k = 1/5$ and $\theta = 5$. Perform 5 simulations, each run to 1,000,000 time units, and report the simulated mean number-in-system, averaged over the 5 simulations.

      (a) What is the relative error in each of the 4 cases?

      (b) Does the relative error increase or decrease as ρ increases?

      (c) Does the relative error increase or decrease as the variability of the service time distribution increases?

3. Consider two GI/GI/1 queues in tandem. Assume that arrivals to the first station follow a Poisson process with rate α, that service times at the first station are exponentially distributed with mean 1, and that service times at the second station are deterministic with mean 1.

   (i) What is the appropriate two-dimensional RBM approximation for this system? Specify its drift, covariance matrix, and reflection matrix.

   (ii) Does the RBM you specified in (i) above have a product form stationary distribution? If so, specify the stationary distribution, and approximate the mean number of customers at each station.

   (iii) Simulate the system using ExtendSim when $\alpha = 0.95$. Perform 5 simulations, each run to 1,000,000 time units, and report the simulated mean number of customers at each station, averaged over the 5 simulations. If you could make an approximation in part (ii), how does your answer compare to that approximation?